Some notes on Chapter 15 (multivariable optimization problems)

1. **Overview of the main types of optimization problems:**
   a. 15.1 Optimize *f* on an unbounded domain
   b. 15.2 Optimize *f* on a closed, bounded domain by creating a "combined function" and finding its critical points.
   c. 15.3 Optimize *f* on a closed, bounded domain using Lagrange Multipliers

2. **15.1 Optimize *f* on an unbounded domain**
   a. There are two types of critical points
      i. Grad f = **0**
      ii. Grad f is undefined
   b. 2^{nd} derivatives test
      i. Only applies to grad f = **0** type critical points
      ii. Only tells you if a point is a <u>local</u> extremum. It does not tell you if the point is also a global extremum.

3. **15.2 Global extrema, application problems and bounded (constrained) domains**
   a. Theorems regarding <u>global</u> extrema
      i. If f is <u>quadratic</u> then a local extremum is also a global extremum.
      ii. If the domain is <u>closed and bounded</u> and f is continuous on the domain, then
         1. There must be a global max and a global min.
         2. These can either occur (a.) at a critical point or (b) at a point on the boundary of the domain (a.k.a. Lagrange point)
      iii. If the domain is unbounded, then
         1. There may or may not be a global max/min.
         2. If there is a global max/min it occurs either (a.) at a critical point, or (b.) as r approaches infinity (i.e. as you go outward in some direction forever).

4. **15.3 The Method of Lagrange Multipliers**
   a. Two types: **g=c** (constraint is a curve), **g<=c** (constraint is a region with the boundary g=c).
   b. Solve only <u>one</u> equation for lambda.
   c. **Look out for extra cases!!!**
      i. Ex.1 $2\lambda x = y \longrightarrow case\ \ 1: \lambda = \dfrac{y}{2x},\ \ case\ \ 2: x = 0$
      ii. Ex.2 $\lambda(y-3) = 4x+1 \longrightarrow case\ \ 1: \lambda = \dfrac{4x+1}{y-3},\ \ case\ \ 2: y = 3$
   d. **Don't use 2^{nd} derivatives test with Lagrange problems**, because…
      i. 2^{nd} derivatives test does not work at Lagrange points
      ii. 2^{nd} derivatives test only tells you if a grad f=**0** type critical point is a <u>local</u> extremum (not if a point is a **global** extremum).
   e. **Do** Evaluate f at all Lagrange points (and if g**<=**c, at any critical points). The highest function value is the global max, and the lowest function value is the global min.